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INELASTIC AND LARGE DEFORMATION ANALYSES OF PLANE TRUSSES

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Abstract

This paper is concerned with inelastic and large deformation analyses of plane trusses. The effects of member buckling and yielding on ultimate strengths of plane trusses are investigated with large deformation of members. Material nonlinearity is considered by introducing plastic hinge concept and plastic hinges are assumed to be formed where the cross-sectional forces satisfy the plasticity criterion which is expressed by LRFD equations. On the other hand, equilibrium equations are computed using the deformed shape of the plane truss for accounting large deformation effects. In order to reveal the accuracy and efficiency of the analytical method used in the study, various numerical examples from the literature are examined accounting both large deformation and material nonlinearity effects.

Key Words: Inelastic Analysis, Large Deformation Analysis, Plane Truss

1. Introduction

Truss structures are one of the most efficient structural systems that are used in today's design. Trusses can sustain considerable loads and behave favorably by using a small amount of materials. Since the beginning of the commercial use, truss systems have been increasingly popular, especially in large open areas with few or no intermediate supports as shown in Figure 1. Over the years, they have gained more attention for their pleasing appearance, light weight, easy fabrication and rapid erection. Thousands of successful structural truss applications now exist all over the world covering stadiums, public halls, exhibition centers, airplane hangers and many other engineering structures. Also, the interest in the analysis of this type of structures lies both in the fact that their use is expanding beyond the traditional one, such as electric towers, windmills, cranes and suspension bridges.

Figure 1. Truss Structures

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Plane truss systems with pinned joints are frequently used in structural engineering applications whose ultimate loading capacity cannot be solely determined from conventional linear analysis. This problem leads an increasing attention to the nonlinear analysis of truss structures whose individual elements can only resist axial forces. In recent years, there is an increasing awareness of the quality and capability of analysis techniques that explicitly consider both material nonlinearity and large deformation effects for estimating the structural response up to maximum load limit states. Some of the studies carried out in this field consider only a nonlinear behavior due to large deformations, while material law is assumed to be linearly elastic [1, 2]. In the analysis of truss structures with pinned joints it is very often necessary to consider loading conditions for which the deflections are large enough to cause significant changes in the geometry of the structure so that the equations of equilibrium must be formulated for the deformed configuration. The solution for deflections and stresses is generally presented as step-by-step matrix method based on load increments which are particularly suitable for computer programming [3]. In addition, large deformation analysis provides eigenvalue equations and global structural stability can also be checked in each step. The theoretical results of the large deformation analysis are compared with the exact analytical results in many studies [3, 4]. In comparison with the large amount of research on the large deformation behaviors of plane trusses, relatively little work has been conducted on the effects of member buckling and yielding [5]. The different types of inelastic analysis models may be generalized into two main groups as elastic – plastic hinge and plastic zone or distributed plasticity [6]. This generalization is based on the degree of refinement in representation of yielding effects. The elastic – plastic hinge model is the simplest approach while the plastic zone model exhibits the greatest refinement [7]. However, in recent years elastic – plastic hinge model become popular since it requires less effort and is less costly than plastic zone model. Elastic – plastic hinge analysis does not represent the distributed plasticity within individual members of the structure but makes use of concentrated plastic hinges to approximate the inelastic behavior of the members in a structure. Thus, this method is computationally more efficient and economical than the plastic zone approach.

In the present study, inelastic and large deformation analyses of plane truss structures are investigated and constitutive laws that consider these effects are outlined for a truss element. In order to account large deformation in analysis procedure large deformation matrix is formed and the equations of equilibrium are formulated with the deformed configuration. Besides, elastic – plastic hinge assumption is used for formulation of inelastic modeling. Elastic – plastic hinge modeling method reduces the complexity of the problem, and minimizes the time required in the solution procedure. In numerical examples, ultimate load capacities of plane truss structures under sustained loading conditions are calculated using the concepts of member buckling and yielding with accounting large deformation of the structures. The ultimate responses and structural behavior of several plane trusses from the literature are examined for determining the accuracy of the proposed method in the study.

2. Large Deformation of Truss Members in Local Coordinates

Numerous papers and many structural analysis programs for digital computers are available for analyzing general types of planar truss structures using conventional linear analysis methods [8, 9]. These methods generally follow the routine for calculating the member stresses and joint deflections in order to check the design code safety limits for complex structures. In last decades, these conventional methods are improved to be employed efficiently for various engineering applications with the developments in computer technology and in computing methods [10, 11]. Additionally, it becomes possible to account large deformations and structural instabilities in the structural analysis procedure. In linear small deformation analysis, equilibrium is computed for the undeformed, original configuration of the truss. On the other hand, in large deformation analysis, equilibrium is computed using the deformed shape of the truss.

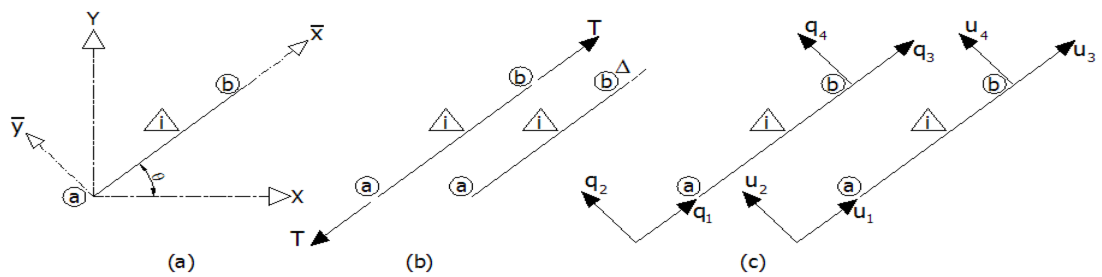

Figure 2. Truss Member: (a) Global and Local Axes, (b) Basic Forces and Displacements, (c) Local Forces and Displacements

In order to calculate large deformation effects in structural analysis; a truss member is selected and the joints of the member ends are termed as a and b, respectively. Figure 2 shows the truss member and its relations with the local and global axes. In large deformation analysis, we assume that the member stiffness matrix may be separated into an elastic part $[k_E]$ plus a geometric part $[k_G]$ which accounts the effects of large deformation. The first matrix is also known as the conventional elastic stiffness matrix $[k_E]$ and it can commonly be recognized from structural analysis textbooks [12]. The elastic stiffness matrix $[k_E]$ is simply given in local coordinates with equation (1) [12]:

$$[k_E] = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

The second matrix which accounts the large deformation effects is derived using the deformed shape of the truss member. If we define an angle θ as the counter clockwise inclination of the deformed bar with respect to global coordinate X and write the relation between local forces and basic forces from Figure 2, we can easily obtain the relationship as given with the equation (2).

$$\{q_1 \quad q_2 \quad q_3 \quad q_4\}^T = T\{-\cos\theta \quad -\sin\theta \quad \cos\theta \quad \sin\theta\}^T \tag{2}$$

The angle θ may be determined from the geometry of the deformed shape using the local displacements as in equation (3) and equation (4):

$$\sin\theta = \frac{1}{L}(u_4 - u_2) \tag{3}$$

$$\cos\theta = \frac{1}{L}(L_0 + u_3 - u_1) = \frac{1}{L}(u_3 - u_1) + \frac{L_0}{L} \approx 1 \tag{4}$$

where L_0 is the original length of the bar and L is the deformed length of the bar.

If we substitute the expressions for sinθ and cosθ into the equilibrium equation (2) with writing the expressions in matrix form, we can obtain the equation (5) where we use the approximation that L_0 is equal to L.

$$\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} = \begin{Bmatrix} -T \\ 0 \\ T \\ 0 \end{Bmatrix} + \frac{T}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \tag{5}$$

If the bar deforms only along its original direction, then $u_2 = u_4 = 0$, $u_3 - u_1 = \Delta$, and $T = EA\Delta/L$ or $T = EA(u_3 - u_1)/L = -q_1 = q_3$ can easily be determined. Therefore, the vector $\{-T \quad 0 \quad T \quad 0\}^T$, in equation (2) can be calculated from the elastic stiffness matrix, and the equation (5) may be formed as equation (6):

$$[k] = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \frac{T}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \tag{6}$$

where the first matrix is the elastic stiffness matrix $[k_E]$ and the second matrix is the large deformation matrix $[k_G]$.

3. Inelastic Modeling of Truss Members

In this study, inelastic behavior of plane truss structures is modeled by using elastic–plastic hinge concept. It is assumed that inelasticity in truss members is concentrated at plastic hinges. The elastic–plastic hinge analysis is a series of analysis in which a plastic hinge is formed at the location of maximum axial force. The formation of the plastic hinge results in a reduction of one degree of indeterminacy from the structure. If the structure is determinate, it becomes unstable and then the ultimate load is obtained. If the structure is indeterminate, it is equivalent to a new and simpler structure in which a real hinge is used to replace the plastic hinge and a constant plastic force is applied at the location. Next, another analysis is performed on the new structure. The new plastic hinge can be located and a newer structure can be obtained. This process is performed until a sufficient number of plastic hinges are formed or buckling occurs and the original structure is transformed into a failure mechanism.

The inelastic analysis has several benefits over the elastic analysis because one of the important properties of steel, ductility is fully utilized. Plastification of steel is the process of yielding of steel fibers causing the change in stress distribution on a cross section as the axial force increased. Furthermore, the sequence of formation of plastic hinges, the overstrength factor associated with the formation of each plastic hinge, and member force distributions in the truss structure between each plastic hinge formation and the collapse load of the structure is provided.

3.1 Ultimate Strength of Truss Members

In this study, since LRFD is a generally known and frequently used standard in the steel structures, the ultimate strength of a truss member is determined by the LRFD equations [13]. In the analysis process, if the axial force of the member is tension, the ultimate strength is provided with equation (7) [13].

$$P_u = \phi_t F_y A \tag{7}$$

where F_y is specified minimum yield stress of steel, ϕ_t is resistance factor for tension, A is cross sectional area.
It is generally more complicated to analyze when the axial force in the member is compression. If the axial force is compression, the slenderness of the section which is given in equation (8) becomes a key factor and ultimate compression force is calculated as given in equation (9) or in equation (10) [13].

$$\lambda_c = \frac{Kl}{r\pi}\sqrt{\frac{F_y}{E}} \tag{8}$$

$$P_u = \phi_c Q\left(0.658^{Q\lambda_c^2}\right)F_y A \qquad \text{for} \qquad \lambda_c\sqrt{Q} \le 1.5 \tag{9}$$

$$P_u = \phi_c \left[\frac{0.877}{\lambda_c^2}\right]F_y A \qquad \text{for} \qquad \lambda_c\sqrt{Q} > 1.5 \tag{10}$$

where F_y is specified minimum yield stress of steel, Q is reduction factor for local buckling, A is gross cross sectional area, ϕ_c is resistance factor for compression, L is unbraced length, r_s is radius of gyration about the plane of buckling, λ_c is slenderness parameter, K is effective length factor and E is modulus of elasticity.

4. Numerical Examples

In this study, a practical incremental method which is straightforward in concept and implementation is used in the analysis in order to investigate inelastic and large deformation analyses of planar truss structures. Since effects of member buckling and yielding on ultimate strengths are accounted using LRFD equations and large deformation analysis is based on large deformation matrix, the validity and computational efficiency of the results are examined with the numerical examples which can be followed from the literature. In all numerical examples, the direction of the force is accounted as positive and the load–deflection curves obtained with respect to this assumption.

4.1 Three–Element Truss

A two–dimensional three–element truss which is generally used in basic engineering problems is subjected to a vertical load at point A and the geometry of the three–element truss is shown in Figure 3. In the reference study, W14×82 steel section is used for all members of the plane truss system and the stress–strain relationship is assumed to be elastic–perfectly plastic accounting yield stress 250 MPa and elastic modulus 200000 MPa [14]. In order to investigate the efficiency of the analysis method, results from the proposed and reference analysis are compared.

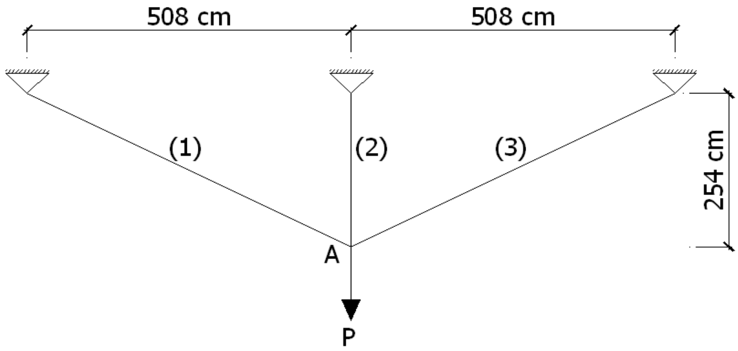

Figure 3. Three-Element Truss

The load–vertical displacement result of the proposed analysis is shown in Figure 4. In this study, material nonlinearity is modeled by using LRFD equations and elastic–plastic hinge assumption while the reference study modeled material nonlinearity using Column Research Council tangent modulus equations and calculated gradual changes of the stiffness using the advanced analysis method [14].

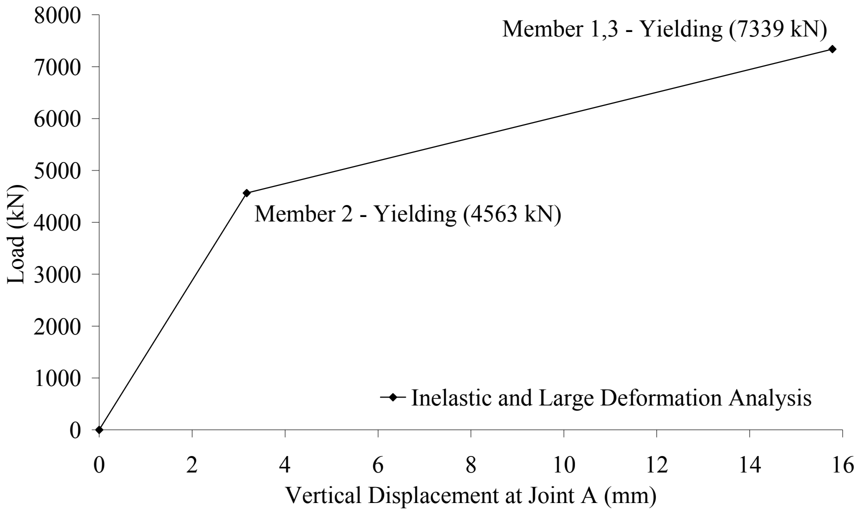

Figure 4. Load–Vertical Displacement of Three-Element Truss

This study and the reference study calculate the ultimate loads of 7339 kN and 7365 kN, respectively [14]. The difference in the ultimate loads between two studies is less than 0.3%. In order to show the analysis practice, each member failure steps are summarized in Table 1.

Table 1. Member Failure Loads of Three–Element Truss

	This Study		**Reference Study [14]**	
	1st Failure	**2nd Failure**	**1st Failure**	**2nd Failure**
Load (P)	4563 kN (Yielding at Member 2)	7339 kN (Yielding at Members 1,3)	4557 kN (Yielding at Member 2)	7365 kN (Yielding at Members 1,3)

4.2 Truss with Double Braced Panel

A plane truss with double braced panel is subjected to a horizontal concentrated load at point A and given in Figure 5. W14×82 is used for all members and the stress–strain relationship is assumed to be elastic–perfectly plastic with yield stress of 250 MPa and elastic modulus of 200000 MPa [14].

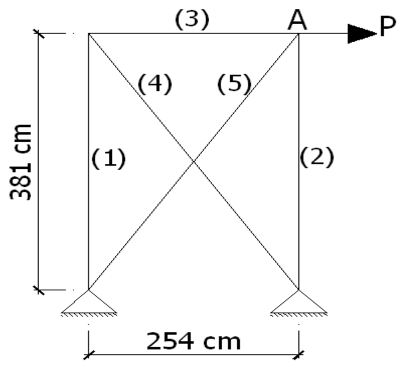

Figure 5. Truss with Double Braced Panel

The load–horizontal displacement result from the proposed analysis is given in Figure 6 and beyond this point displacement values increases abruptly that indicates system failure.

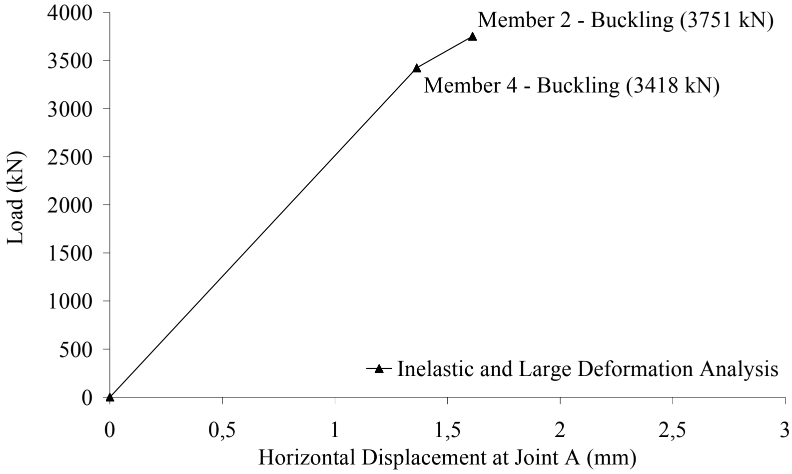

Figure 6. Load–Horizontal Displacement of Truss with Double Braced Panel

The member failure loads of the proposed and the reference analysis are given in Table 2 [14].

Table 2. Member Failure Loads of Truss with Double Braced Panel

	This study		**Reference Study [14]**	
	1st Failure	**2nd Failure**	**1st Failure**	**2nd Failure**
Load (P)	3418 kN (Buckling at Member 4)	3751 kN (Buckling at Member 2)	3418 kN (Buckling at Member 4)	3751 kN (Buckling at Member 2)

4.3 Pratt Truss

The Pratt shape truss is a well-known type of truss structure which was patented by two Boston railway engineers [15]. Pratt truss is generally used for medium spans and this type of truss structure remained popular even as wood gave way to iron, and even still as iron gave way to steel. In this study, a type of Pratt steel truss which was used by the reference study is investigated in order to check the validity of the proposed analysis method [16]. The geometry and loading of the Pratt truss is given with Figure 7. The Pratt truss has an advantage that the diagonals except end posts are all in tension for uniform loads at the joints of the lower chords. A square structural tube of 10x10x5/8 for the upper chords and vertical members and W10×49 section for lower chords and diagonals are used in the reference study [16]. Also, A46 steel is used for compression members and A36 steel is used for tension members, respectively [16].

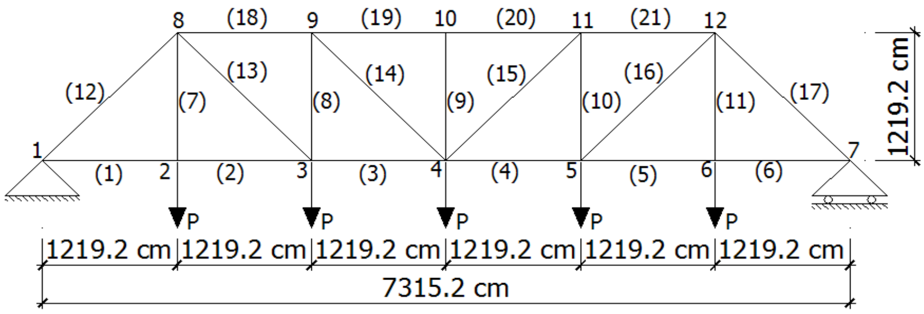

Figure 7. Pratt Truss

The load–horizontal displacement result from the proposed analysis is given in Figure 8.

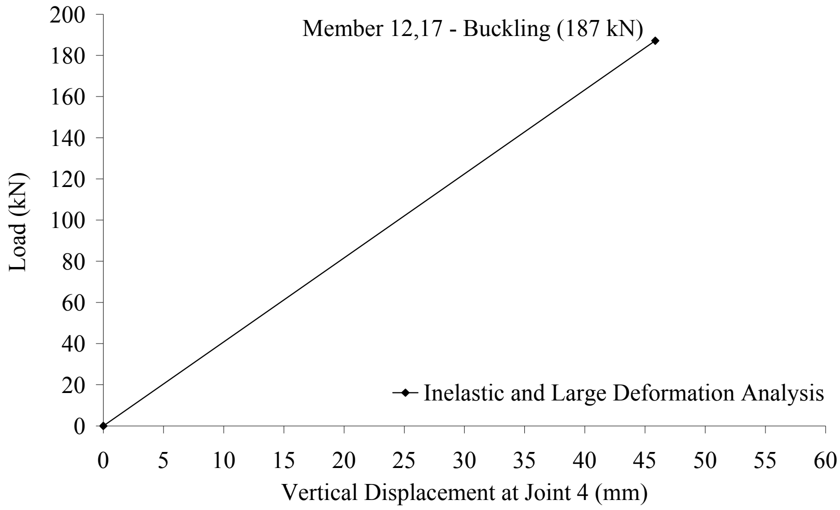

Figure 8. Load – Vertical Displacement of Pratt Truss

The member failure loads of the proposed and the reference analysis are given in Table 3 [15].

Table 3. Member Failure Loads of Pratt Truss

	This study	**Reference Study [15]**
	Failure	**Failure**
Load (P)	187 kN (Buckling at Member 12, 17)	191 kN (Buckling at Member 12, 17)

4.4 Arch Truss

An arch truss which was analyzed in the reference study is shown in Figure 9. In order to check the validity of the proposed analysis method of the current study, all the joints are assumed to be pinned and only the in-plane behavior is considered as in the reference study [17].

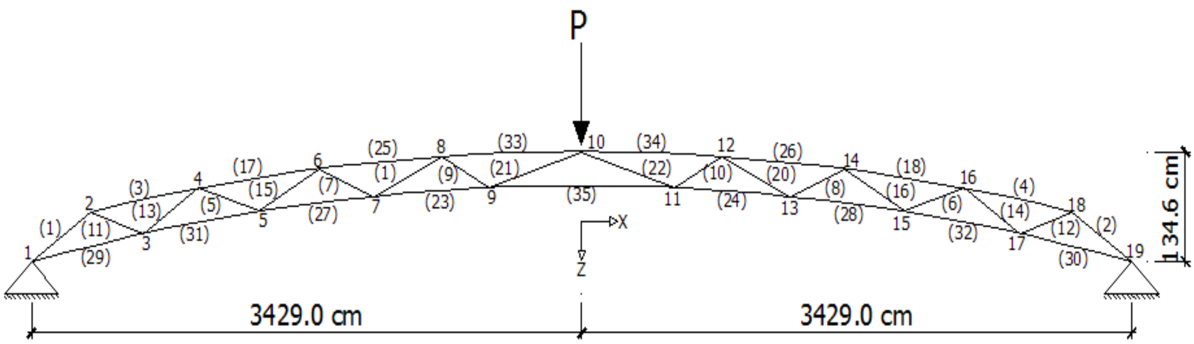

Figure 9. Arch Truss

For ease of modeling, the coordinates of the joints as well as member size making up the truss are tabulated in Table 4 and Table 5. The modulus of elasticity for the material is taken as 7.03 x 10^5 kg/cm^2 and the cross section of all members is solid circular [17].

Table 4. Coordinates of Arch Truss

Node Number	X Coordinate (cm)	Z Coordinate (cm)
1,19	± 3429	0
2,18	± 3048	50.65
3,17	± 2667	34.75
4,16	± 2286	83.82
5,15	± 1905	65.30
6,14	± 1524	110.85
7,13	± 1143	87.99
8,12	± 762	128.50
9,11	± 381	100.05
10	0	134.60

Table 5. Cross Sectional Area of Members

Member Number	Cross Sectional Area (cm^2)
1 – 10, 35	51.61
11, 12	64.52
13 – 16	83.87
17, 18	96.77
19 – 22	103.23
23, 24	161.29
25, 26	193.55
27, 28	258.06
29 – 32	290.32
33, 34	309.68

The load–vertical displacement curve of the arch truss is given in Figure 10.

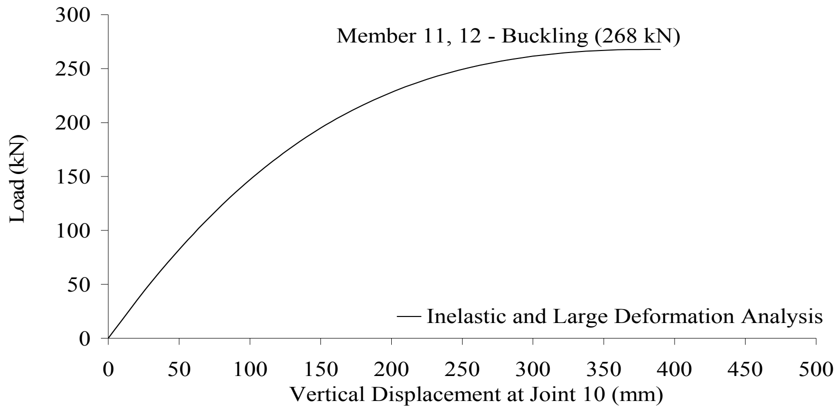

Figure 10. Load–Vertical Displacement of Arch Truss

The ultimate load capacity of the analysis results of this study and reference study is given in Table 6 [17].

Table 6. Member Failure Loads of Arch Truss

	This study	Reference Study [17]
	Failure	**Failure**
Load (P)	268 kN (Buckling at Member 11, 12)	267 kN (-)

5. Conclusion

Following the advances with the computer–based analysis technique, structural analysis is tending towards more on the overall system response and less on individual member response. Inelastic and large deformation analyses of planar trusses are foremost analyze methods that incorporate the aspects of engineering involving mechanics, material behavior and computing.

In this study, the effects of member buckling and yielding on ultimate strengths of planar trusses are investigated accounting large deformation effects. The load–displacement relationship comprising the key factors influencing plane truss behavior is described. The concluding remarks may be outlined as follows:

1. Practical methods for inelastic and large deformation analyses of plane truss structures are outlined.
2. The material nonlinearity is implemented by using LRFD equations and the deformed shape of the planar trusses is considered in the structural analysis by accounting large deformation effects.
3. Numerical examples have provided similar results with the previous studies from the literature and also showed the ease and convenience of the proposed analysis method.
4. When it is compared with the linear elastic analysis methods, inelastic and large deformation analyses of planar trusses provide more information such as the locations and sequences of formation of plastic hinges, inelastic redistribution of internal forces, failure mechanism and over strength factor of the structural system.
5. This study provides a structural analysis approach which is consistent with the general concepts of the performance based–design. The analyses described in the study are user–friendly for computer–based analysis procedures.

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